GLG 490/598 - Subduction Zones **Fall, 2003**
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Problem Set #2 - Global plate motions
Due Thursday, September 4

The motion of a rigid plate across the surface of the Earth is completely described by a pole of rotation (the Euler pole) and the angular velocity about the pole of rotation. The global plate model NUVEL-1 is based on 1122 data (spreading rates, transform fault azimuths, and earthquake slip vectors) and is accurate to within 3 mm/yr (see DeMets et al., 1990, *Geophys. J. Int.*, v. 101, p. 425-478). In this problem set we will use the NUVEL-1 Euler vectors (= Euler pole + angular velocity) to determine the relative plate motions along several interesting plate boundaries.

(A) Cascadia subduction zone - Handout A

(i) Using the NUVEL-1 Pacific - Juan de Fuca plate Euler vector, calculate the relative motion (velocity and azimuth) of the Juan de Fuca plate with respect to the Pacific plate at:

Point (1) 42 °N, 127.1 °W
Point (2) 47 °N, 129.1 °W

(ii) The Euler vectors describing the relative motion of the North American plate and the Juan de Fuca plate can be determined from adding the Pacific - Juan de Fuca Euler vector to the North America - Pacific Euler vector. The resulting North America - Juan de Fuca Euler vector is: 20.7 °N, 112.2 °W, 0.80 degrees $m.y.^{-1}$. Using this Euler vector, calculate the relative motion (velocity and azimuth) of the Juan de Fuca plate with respect to North America at:

Point (3) 41 °N, 124.8 °W
Point (4) 44 °N, 125.3 °W
Point (5) 47 °N, 125.9 °W
Point (6) 50 °N, 128.3 °W

(iii) On handout A, plot the relative plate motion at each of the six points, using a vector pointing in the correct direction and labeling the vector with the rate in millimeters per year (= km $m.y.^{-1}$).

(iv) How does the spreading rate along the Juan de Fuca (and Gorda) ridge vary from north to south? How does the Cascadia subduction rate vary from north to south?

(B) San Andreas Fault - Handout B

(i) Using the NUVEL-1 Pacific - North America Euler vector, calculate and plot the relative motion between the two plates at Parkfield located on the San Andreas fault.

(ii) Detailed trenching studies (which yield an average slip rate for the past 13,250 years) and recent geodetic surveys (which yield an average slip rate for the past 20 years) both indicate that the average slip rate along the central San Andreas is approximately 35 mm yr^{-1}. How does the Pacific - North America slip rate calculated using NUVEL-1 compare to the San Andreas slip rate determined from trenching and geodetic surveys? What possible reasons might explain the discrepancy?

(C) Aleutian subduction zone - Handout C

(i) Using the NUVEL-1 Pacific - North America Euler vector, calculate and plot the subduction rate of the Pacific plate beneath North America at:

Point (1) 56.4 °N, 150 °W
Point (2) 53.2 °N, 160 °W
Point (3) 50.8 °N, 170 °W
Point (4) 49.7 °N, 180 °
Point (5) 52.0 °N, 170 °E

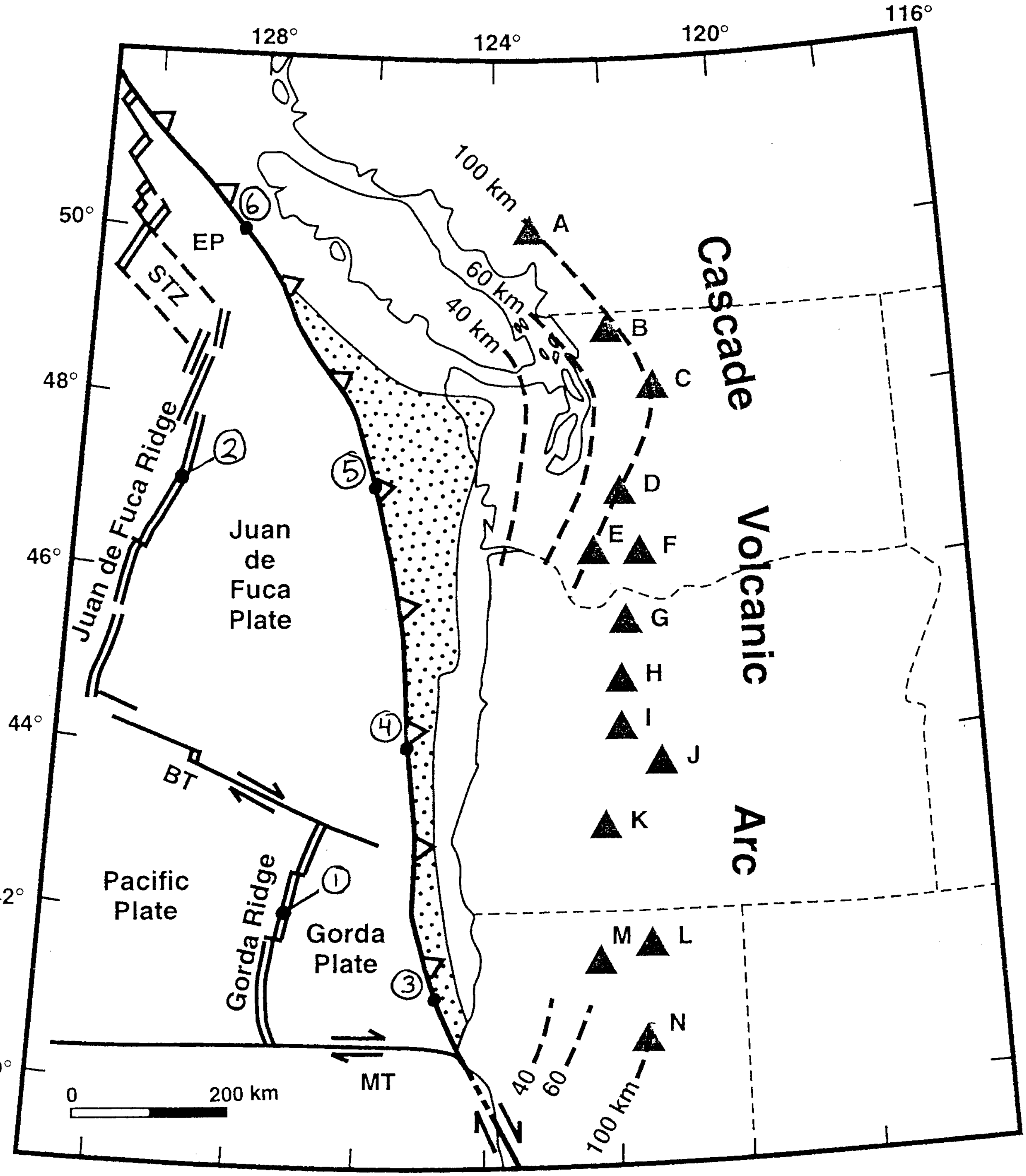
(ii) How does the subduction rate and direction vary from east to west? What changes would you expect in the tectonic setting (e.g., earthquakes, magmatism) from east to west?

Table 1. NUVEL-1 Euler vectors (Pacific plate fixed).

Plate	Latitude °N	Longitude °E	ω (deg–m.y.$^{-1}$)	ω_x	ω_y (radians–m.y.$^{-1}$)	ω_z
Africa	59.160	–73.174	0.9695	0.002511	–0.008303	0.014529
Antarctica	64.315	–83.984	0.9093	0.000721	–0.006841	0.014302
Arabia	59.658	–33.193	1.1616	0.008570	–0.005607	0.017496
Australia	60.080	1.742	1.1236	0.009777	0.000297	0.016997
Caribbean	54.195	–80.802	0.8534	0.001393	–0.008602	0.012080
Cocos	36.823	–108.629	2.0890	–0.009323	–0.027657	0.021853
Eurasia	61.066	–85.819	0.8985	0.000553	–0.007567	0.013724
India	60.494	–30.403	1.1539	0.008555	–0.005020	0.017528
Nazca	55.578	–90.096	1.4222	–0.000023	–0.014032	0.020476
North America	48.709	–78.167	0.7829	0.001849	–0.008826	0.010267
South America	54.999	–85.752	0.6657	0.000494	–0.006646	0.009517
Additional Euler Vectors (Pacific Plate Fixed)						
Juan de Fuca	35.0	26.0	0.53	0.00681	0.00332	0.00531
Philippine	0.	–47.	1.0	0.0119	0.0128	0.000

Each named plate moves counterclockwise relative to the Pacific plate. The Juan de Fuca-Pacific 3.0 Ma Euler vector is taken from Wilson (1988) and the Philippine-Pacific Euler vector is taken from Seno *et al.* (1987).

DeMets et al. (1990)



Accretionary complex

after Wells and Weaver (1994) and Wilson (1993)

HANDOUT A

HANDOUT B

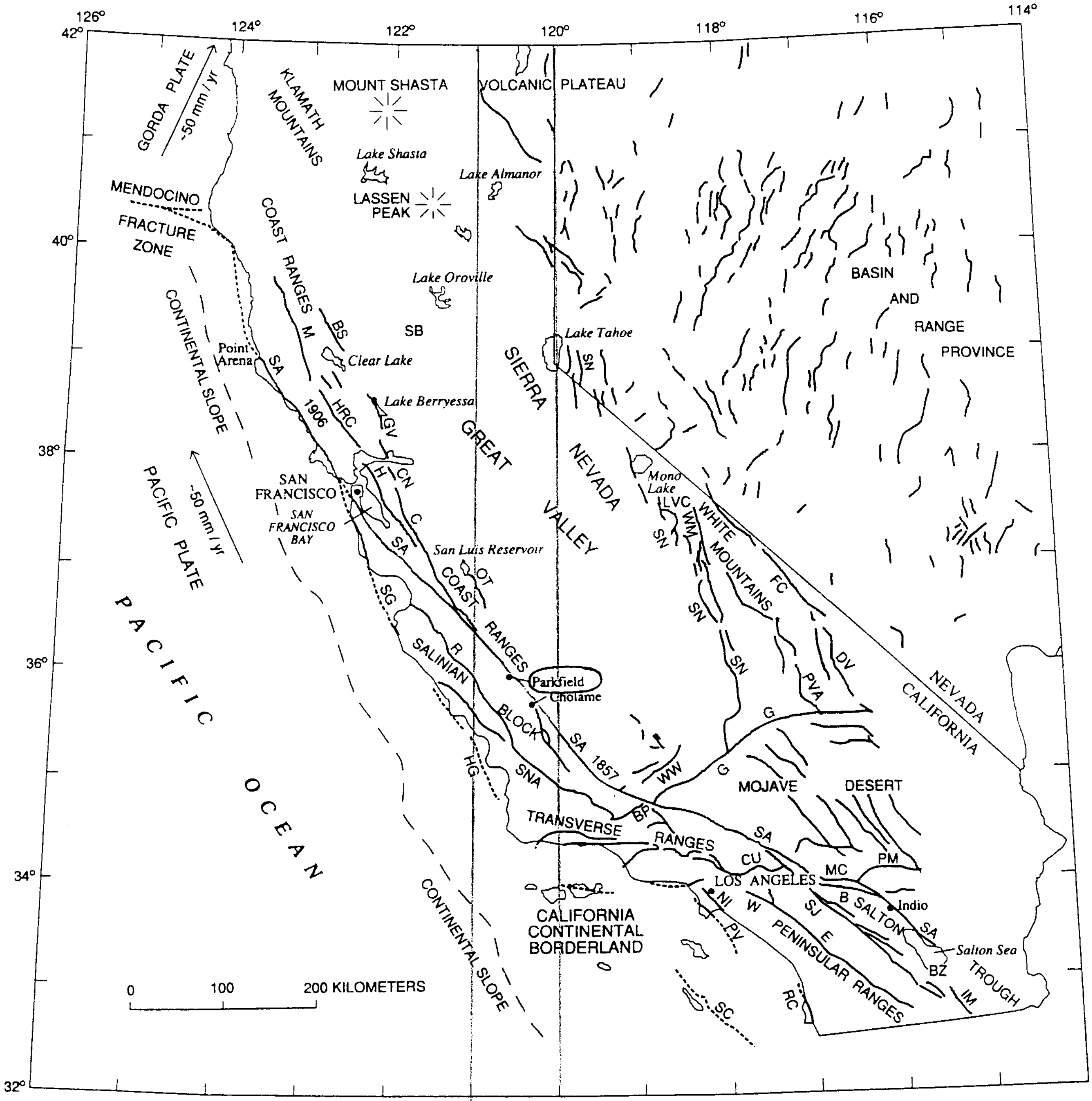


FIGURE 5.3.—Place names and faults most commonly used in text (see front of book for more complete maps of place names and faults). Faults (dotted where concealed): B, Banning; BP, Big Pine; BS, Bartlett Springs; BZ, Brawley seismic zone; C, Calaveras; CN, Concord; CU, Cucamonga; DV, Death Valley; E, Elsinore; FC, Furnace Creek; G, Garlock; GV, Green Valley; H, Hayward; HG, Hosgri; HRC, Healdsburg-Rodgers Creek; IM, Imperial; LVC, Long Valley caldera; M, Maacama; MC, Mission Creek; NI, Newport-Inglewood; OT, Ortigalita; PM, Pinto Mountain; PV, Palos Verdes; PVA, Panamint Valley; R, Rinconada; RC, Rose Canyon; SA, San Andreas; SC, San Clemente Island; SG, San Gregorio; SJ, San Jacinto; SN, Sierra Nevada; SNA, Sur-Nacimiento; W, Whittier; WM, White Mountains; WW, White Wolf. Arrows and numbers indicate direction and amount of motion, respectively, of Pacific and Gorda plates with respect to North American plate to the east; red lines indicate 1857 and 1906 ruptures of San Andreas fault.

Hill et al., 1990, USGS Prof. Pap. 1515

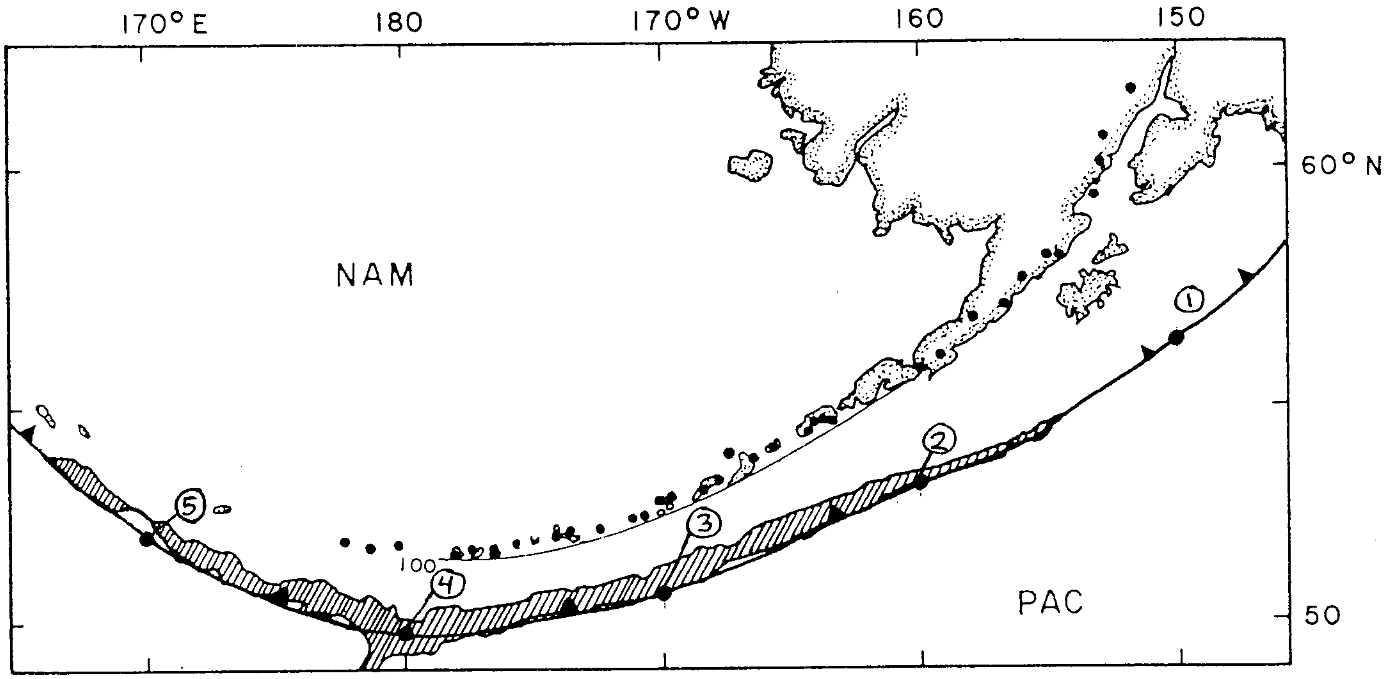


Fig. 2.2.13. Aleutians-Alaska. *Shaded area* encloses > 3000 fathom depths. Seismicity contour from Isacks and Molnar (1971). For more precise volcano locations see Coats (1962)

Gill (1981)

HANDOUT C

taken from Fowler (1990)

2.3 Rotation Vectors and Rotation Poles

To describe motions on the surface of a sphere we use *Euler's 'fixed point' theorem*, which states: 'The most general displacement of a rigid body with a fixed point is equivalent to a rotation about an axis through that fixed point'.

Taking a plate as a rigid body and the centre of the earth as a fixed point, we can restate this theorem: 'Every displacement from one position to another on the surface of the earth can be regarded as a rotation about a suitably chosen axis passing through the centre of the earth'.

This restated theorem was first applied by Bullard et al. (1965) in their paper on continental drift, in which they describe the fitting of the coastlines of South America and Africa. The 'suitably chosen axis' which passes through the centre of the earth is called the *rotation axis*, and it cuts the surface of the earth at two points called the *poles of rotation* (Fig. 2.8a). These are purely mathematical points and have no physical reality, but their positions describe the directions of motion of all points along the plate boundary. The magnitude of the angular velocity about the axis then defines the magnitude of the relative motion between the two plates. Because angular velocities behave as vectors, the relative motion between two plates can be written as $\boldsymbol{\omega} = \omega \mathbf{k}$, where $\mathbf{k}$ is a unit vector along the rotation axis and ω is the angular velocity. The sign convention used is that a rotation which is clockwise (or right-handed) when viewed from the centre of the earth along the rotation axis is positive. Viewed from outside the earth, a positive rotation is anticlockwise. Thus, one rotation pole is positive and the other is negative (Fig. 2.8b).

Consider a point X on the surface of the earth (Fig. 2.8c). At X the value of the relative velocity v between the two plates is

$$v = \omega R \sin \theta \tag{2.3}$$

where θ is the angular distance between the rotation pole P and the point X, R is the radius of the earth. Thus, the relative velocity is zero at the rotation poles, where $\theta = 0°$ and $180°$, and has a maximum value of ωR at $90°$ from the rotation poles. This factor of $\sin \theta$ means that the relative motion between two adjacent plates changes with position along the plate boundary, in contrast to the earlier examples for a flat earth. If by chance the plate boundary passes through the rotation pole, the nature of the boundary changes from divergent to convergent, or vice versa (Fig. 2.8b). Lines of constant velocity are small circles defined by θ = constant about the rotation poles.

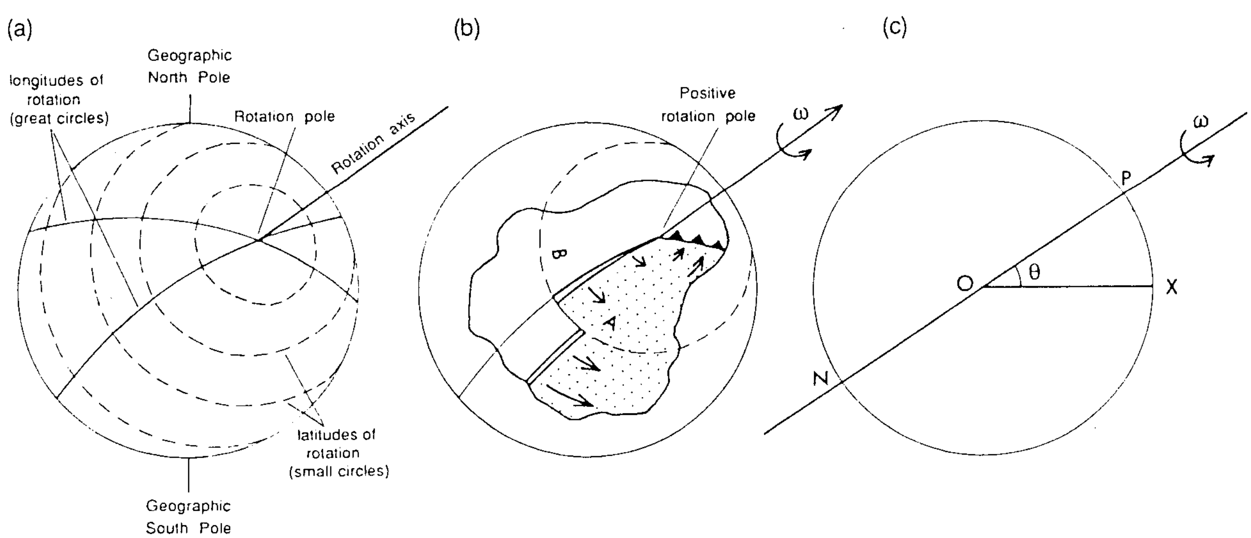


Figure 2.8. The movement of plates on the surface of the earth. (a) The lines of latitude of rotation around the rotation poles are small circles (shown dashed), whereas the lines of longitude of rotation are great circles (i.e., circles with the same diameter as the earth). Note that these lines of latitude and longitude of rotation are *not* the geographical lines of latitude and longitude because the poles for the geographical coordinate system are the North and South poles, not the rotation poles. (b) Constructive, destructive and conservative boundaries between plates A and B. Plate B is assumed to be fixed so that the motion of plate A is relative to plate B. The visible rotation pole is positive (motion is anticlockwise when viewed from outside the earth). Note that the spreading and subduction rates increase with distance from the rotation pole. The transform fault is an arc of small circle (shown dashed) and thus is perpendicular to the ridge axis. As the plate boundary passes the rotation pole, the boundary changes from ridge to subduction zone. (c) Cross section through the centre of the earth O. P and N are the positive and negative rotation poles, and X is a point on the plate boundary.

2.4.2 *Calculation of the Relative Motion at a Plate Boundary*

After the instantaneous rotation pole and angular velocity have been determined for a pair of adjacent plates, they can be used to calculate the

Table 2.2. *Symbols used in calculations involving rotation poles*

Symbol	Meaning	Sign convention
λ_P	Latitude of rotation pole P	°N positive
λ_X	Latitude of point X on plate boundary	°S negative
ϕ_P	Longitude of rotation pole P	°W negative
ϕ_X	Longitude of point X on plate boundary	°E positive
$\mathbf{v}$	Velocity at point X on plate boundary	
v	Amplitude of velocity	
β	Azimuth of the velocity with respect to north N	Clockwise positive
R	Radius of the earth	
ω	Angular velocity about rotation pole P	

direction and magnitude of the relative motion at any point along the plate boundary.

The notation and sign conventions used in the following pages are given in Table 2.2. Figure 2.11 shows the relative positions of the North Pole N, positive rotation pole P and point X on the plate boundary (compare with Fig. 2.8b). In the spherical triangle NPX, let the angles $\widehat{XNP} = A$, $\widehat{NPX} = B$ and $\widehat{PXN} = C$, and let the angular lengths of the sides of the triangle be $PX = a$, $XN = b$ and $NP = c$. Thus, the angular lengths b and c are known, but a is not:

$$b = 90 - \lambda_X \tag{2.4}$$

$$c = 90 - \lambda_P \tag{2.5}$$

Angle A is known, but B and C are not:

$$A = \phi_P - \phi_X \tag{2.6}$$

Equation 2.3 is used to obtain the magnitude of the velocity at point X:

$$v = \omega R \sin a \tag{2.7}$$

The azimuth of the velocity β is given by

$$\beta = 90 + C \tag{2.8}$$

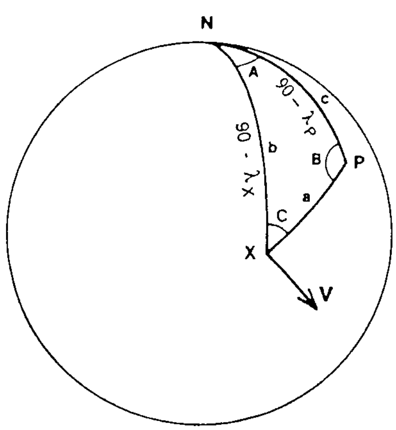


Figure 2.11. Diagram showing the relative positions of the positive rotation pole P and point X on the plate boundary. N is the North Pole. The sides of the spherical triangle NPX are all great circles, the sides NX and NP are lines of geographic longitude. The vector $\mathbf{v}$ is the relative velocity at point X on the plate boundary (note that it is perpendicular to PX). It is usual to quote the lengths of the sides of spherical triangles as angles (e.g., latitude and longitude when used as geographic coordinates).

To find the angles a and C to substitute into Eqs. 2.7 and 2.8, we use spherical geometry. Just as there are cosine and sine rules which relate the angles and sides of plane triangles, there are cosine and sine rules for spherical triangles:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \tag{2.9}$$

and

$$\frac{\sin a}{\sin A} = \frac{\sin c}{\sin C} \tag{2.10}$$

Substituting Eqs. 2.4–2.6 into Eq. 2.9 gives

$$\cos a = \cos(90 - \lambda_X)\cos(90 - \lambda_P) + \sin(90 - \lambda_X)\sin(90 - \lambda_P)\cos(\phi_P - \phi_X) \tag{2.11}$$

This can then be simplified to yield the angle a, which is needed to calculate the velocity from Eq. 2.7.

$$a = \cos^{-1}[\sin\lambda_X \sin\lambda_P + \cos\lambda_X \cos\lambda_P \cos(\phi_P - \phi_X)] \tag{2.12}$$

Substituting Eqs. 2.5 and 2.6 into Eq. 2.10 gives

$$\frac{\sin a}{\sin(\phi_P - \phi_X)} = \frac{\sin(90 - \lambda_P)}{\sin C} \tag{2.13}$$

Upon rearrangement this becomes

$$C = \sin^{-1}\left[\frac{\cos\lambda_P \sin(\phi_P - \phi_X)}{\sin a}\right] \tag{2.14}$$

Therefore, if the angle a is calculated from Eq. 2.12, angle C can then be calculated from Eq. 2.14, and finally, the relative velocity and its azimuth can be calculated from Eqs. 2.7 and 2.8. Note that the inverse sine function of Eq. 2.14 is double-valued.* Always check that you have the correct value for C. ← The easiest way to do this is to use a globe.

Example: calculation of relative motion at a plate boundary

Calculate the present-day relative motion at 28°S 71°W on the Peru–Chile Trench using the Nazca–South America rotation pole given in Table 2.1. Assume the radius of the earth to be 6371 km.

$$\lambda_X = -28°, \qquad \phi_X = -71°$$

$$\lambda_P = 56°, \qquad \phi_P = -94°$$

$$\omega = 7.6 \times 10^{-7}\ \text{deg yr}^{-1} = \frac{\pi}{180} \times 7.6 \times 10^{-7}\ \text{radians yr}^{-1}$$

These values are substituted into Eqs. 2.12, 2.14, 2.7 and 2.8 in that order, giving:

$$a = \cos^{-1}[\sin(-28)\sin(56) + \cos(-28)\cos(56)\cos(-94 + 71)]$$
$$= 86.26° \tag{2.15}$$

$$C = \sin^{-1}\left[\frac{\cos(56)\sin(-94+71)}{\sin(86.26)}\right]$$
$$= -12.65° \tag{2.16}$$

$$v = \frac{\pi}{180} \times 7.6 \times 10^{-7} \times 6371 \times 10^5 \times \sin(86.26)\ \text{cm yr}^{-1} \tag{2.17}$$

* An alternative way to calculate motion along a plate boundary and to avoid the sign ambiguities is to use vector algebra (see Altman 1986 or Cox and Hart 1986, p. 154).

$$= 8.43\ \text{cm yr}^{-1}$$

$$\beta = 90 - 12.65 \tag{2.18}$$
$$= 77.35°$$

Thus, the Nazca Plate is moving relative to the South American Plate at 8.4 cm yr^{-1} with azimuth 77°; the South American Plate is moving relative to the Nazca Plate at 8.4 cm yr^{-1}, azimuth 257° (Fig. 2.2).