

**Problem Set #3 - Temperatures and Shear Stresses in Subduction-Zone Forearcs**

**Due Tuesday, September 16**

The plate boundary separating a subducting plate from the overriding plate is a major thrust fault. Molnar and England (1990, J. Geophys. Res., v. 95, 4833-4956) investigated the thermal structure near major thrust faults and their analytical expressions may be used to gain insight into the thermal structure of subduction zones. The steady-state thermal structure of the hanging wall (upper plate) depends on:

- (1) the magnitude and distribution of three heat sources:
  - (a) conduction from below
  - (b) radioactivity
  - (c) shear heating along the thrust fault
- (2) the rate at which movement of the footwall (lower plate) advectively moves heat
- (3) physical properties of the rocks, particularly thermal conductivity and thermal diffusivity.

Assuming negligible radioactive heating, temperatures *within* the hanging wall may be calculated using analytical expressions derived by Molnar and England (1990):

$$T = \frac{(Q_0 + Q_{sh})z/k}{S} \quad (1)$$

where  $T$  = temperature ( $^{\circ}\text{C}$ ),  $Q_0$  = basal heat flux ( $\text{W}/\text{m}^2$ ),  $Q_{sh}$  = rate of shear heating ( $\text{W}/\text{m}^2$ ),  $z$  = depth (m),  $k$  = thermal conductivity ( $\text{W}/\text{m}\cdot\text{K}$ ), and  $S$  = a divisor that accounts for advection given by:

$$S = 1 + b \sqrt{\frac{V z_f \sin \delta}{\kappa}} \quad (2)$$

where  $b$  = a constant ( $\approx 1$  based on numerical experiments),  $V$  = slip rate (m/s),  $z_f$  = depth to the fault (m),  $\delta$  = angle of subduction, and  $\kappa$  = thermal diffusivity ( $\text{m}^2/\text{s}$ ). The numerator in equation (1) represents the steady-state temperatures in the absence of advection and the denominator,  $S$ , represents the effect of advection (i.e., cooling of the hanging wall resulting from underthrusting).

For a constant shear stress along the thrust fault, the rate of shear heating ( $Q_{sh}$ ) is given by:

$$Q_{sh} = \tau V \quad (3)$$

where  $\tau$  = shear stress (Pa) and  $V$  = slip rate along the fault (m/s).

For the following calculations assume  $k = 2.5 \text{ W/m-K}$  and  $\kappa = 1 \times 10^{-6} \text{ m}^2/\text{s}$  ( $=1 \text{ mm}^2/\text{s}$ ).

(A) Calculate the temperature along the thrust fault as a function of depth (0 to 60 km) for shear stresses of 0, 10, 20, and 50 MPa where  $V = 91 \text{ mm/yr}$ ,  $Q_0 = 0.050 \text{ W/m}^2$ , and  $\delta = 20^\circ$  (appropriate for the NE Japan subduction zone). Hint: Because you are calculating temperatures *along* the thrust fault, set  $z = z_f$  in equation (1).

(B) In a subduction zone that is at thermal steady state (i.e., the thermal structure does not change with time), the P-T path followed by the top of the subducting plate will be the same as the P-T conditions along the thrust fault. Using the results from (A), plot the P-T paths followed by rocks at the top of the subducting plate for the four different shear stresses (0, 10, 20, and 50 MPa). Pressure is related to depth by the relation:

$$P = \rho g z \quad (4)$$

where  $P =$  pressure (Pa),  $\rho =$  density (assume  $2850 \text{ kg/m}^3$ ),  $g =$  gravitational constant ( $9.8 \text{ m/s}^2$ ), and  $z =$  depth (m).

(C) Calculate the two-dimensional thermal structure of the upper plate for shear stresses of 0 and 50 MPa (use the same subduction parameters defined in (A) above). Display your results by plotting isotherms (contour interval =  $100^\circ \text{C}$ ) on the two cross sections provided.

(D) The surface heat flow measured in a subduction zone forearc places an important constraint on the thermal structure and the rate of shear heating. Heat flux ( $Q$ ) is related to the thermal gradient by the relation:

$$Q = k \frac{\partial T}{\partial z} \quad (5)$$

Taking the derivative of equation (1) and assuming a constant shear stress yields the following equation for heat flow:

$$Q = \frac{(Q_0 + \tau V)}{S} \quad (6)$$

which we can solve for shear stress to yield:

$$\tau = \frac{(SQ - Q_0)}{V} \quad (7)$$

Estimate the shear stress along the subduction-zone thrust fault based on the following heat flow observations obtained from different subduction zones:

NE Japan:  $Q_0 = 0.050 \text{ W/m}^2$  (appropriate for 130 Ma incoming lithosphere)  
 $V = 91 \text{ mm/yr}$ ,  $\delta = 20^\circ$   
Observed  $Q = 0.032 \text{ W/m}^2$  where  $z_f = 60 \text{ km}$

Cascadia:  $Q_0 = 0.100 \text{ W/m}^2$  (appropriate for 10 Ma incoming lithosphere)  
 $V = 40 \text{ mm/yr}$ ,  $\delta = 13^\circ$   
Observed  $Q = 0.040 \text{ W/m}^2$  where  $z_f = 60 \text{ km}$

Kermadec:  $Q_0 = 0.060 \text{ W/m}^2$  (appropriate for 10 Ma incoming lithosphere)  
 $V = 61 \text{ mm/yr}$ ,  $\delta = 17^\circ$   
Observed  $Q = 0.030 \text{ W/m}^2$  where  $z_f = 30 \text{ km}$

(E) Briefly discuss the major uncertainties in determining shear stresses along a subduction-zone thrust fault using heat flow observations.

